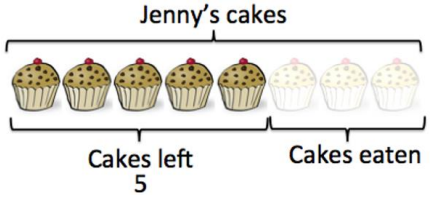
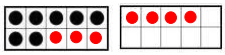
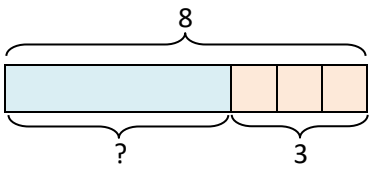
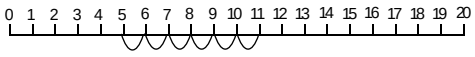
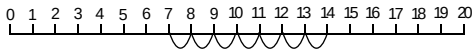
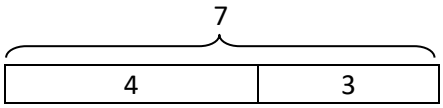
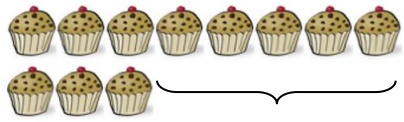
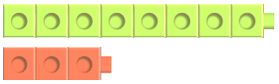
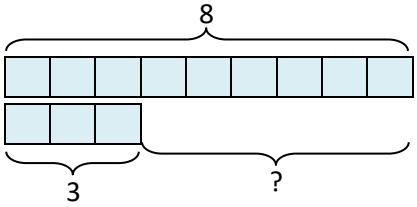
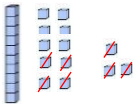
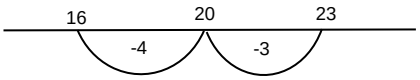
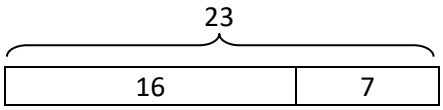
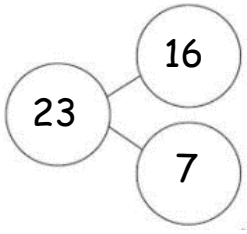
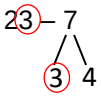
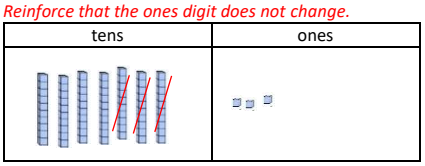
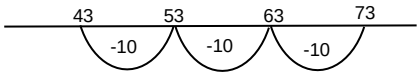
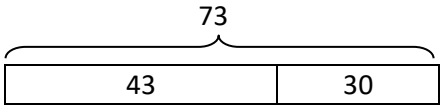
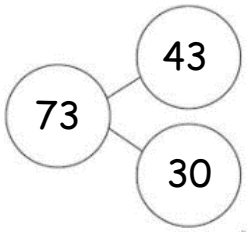
CALCULATION PROGRESSION: SUBTRACTION

Pre-learning 1 Understanding subtraction to 20

concrete	pictorial	abstract
<u>Number bonds to 20</u> Using objects to represent a problem: *Place the total number of objects, then remove some and count the number left over.*  Bead string: *Count back in ones.*  Ten frames: *Make 14 and remove 7 counters.*  Multilink cubes: *Start with the 'whole' and take 'part' away.*  Cuisenaire rods: *Start with the 'whole' and take 'part' away.* 	<u>Number bonds to 20</u> Bar model:  Number line:   Bar model:  	<u>Number bonds to 20</u> 8 – 3 = 5 11 – 6 = 5 14 – 7 = 7 7 – 3 = 4 7 – 4 = 3 Number sentences presented in different ways: 20 - 3 = 17 9 = 17 - 8 7 = □ - 9 □ - 5 = 8
<u>Finding a difference</u> Using objects to represent a problem: *Make both numbers and compare them.*  Multilink cubes: *Make both numbers and compare them.* 	<u>Finding a difference</u> Bar model:  	<u>Finding a difference</u> 8 – 3 = 5 3 + □ = 8

CALCULATION PROGRESSION: SUBTRACTION

Pre-learning 2 TO-O, TO-T & TO-TO

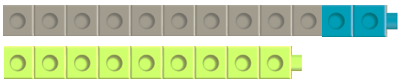
concrete	pictorial	abstract
<u>Two-digit - ones</u> Bead string: *Start with 23, move the beads up to the multiple of ten before, then move the rest.* Base ten blocks: *Make 23, subtract 3 then exchange a ten stick for ten ones to subtract the rest.* 	<u>Two-digit - ones</u> Number line – counting back to the multiple of ten first:  Bar model:  Part-part-whole model:  	<u>Two-digit - ones</u>  *Partition the 7 in this way because taking away 3 leaves a multiple of ten.* 23 – 7 = 23 – 3 – 4 = 16 *Family of four* 16 + 7 = 23 7 + 16 = 23 23 – 7 = 16 23 – 16 = 7
<u>Two-digit - tens</u> Base ten blocks: *Make the starting number, then remove the tens.* *Reinforce that the ones digit does not change.*  **Counting:** Count back in tens from different starting points: 73, 63, 53, 43, … 47, 37, 27, 17, 7, …	<u>Two-digit - tens</u> Drawing base ten:  Number line – counting back in tens:  Bar model:  Part-part-whole model:  	<u>Two-digit - tens</u> 73 – 30 = 43 47 – 40 = 7

Finding a difference

Use this method for comparison problems.

Multilink cubes:

Make both numbers and compare them.

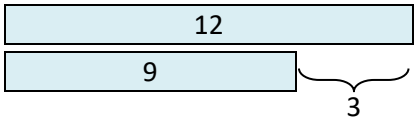


Cuisenaire rods:

Finding a difference

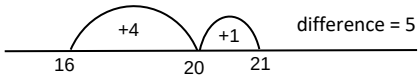
Use this method for comparison problems.

Bar model:



Number line:

Mark both numbers on the number line and count up to find the distance between them.



Finding a difference

Use this method for comparison problems.

$12 - 9 = 3$

$9 + \square = 12$

$21 - 16 = 5$

$16 + \square = 21$

Comparison problem:
Lisa is 16 years old. Her sister is 21 years old.
What is the difference in age between them?

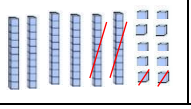
Two-digit - two-digit

Bead string:

Show the first number. Partition the second number and move the tens, then move the ones.

Base ten blocks:

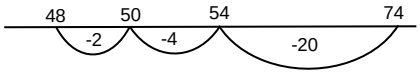
Make the first number. Partition the other number and take away the tens, then take away the ones (exchanging a ten stick for 10 ones if needed).

tens	ones
	

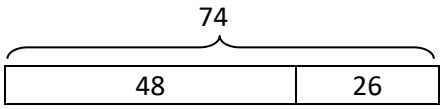
Two-digit - two-digit

Number line:

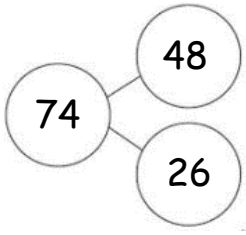
Partition the second number, subtract the tens then subtract the ones.



Bar model:



Part-part-whole model:

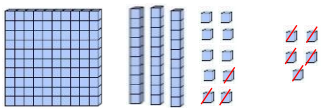
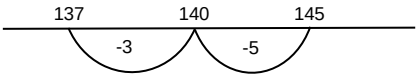
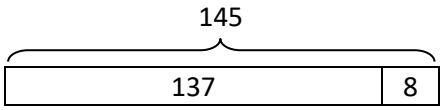
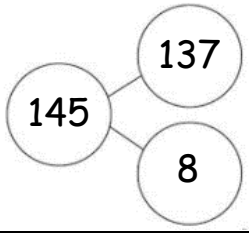
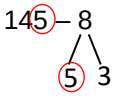
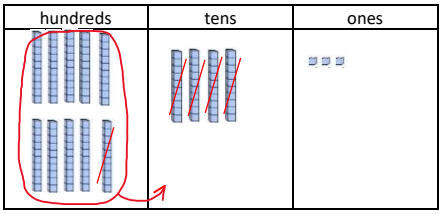
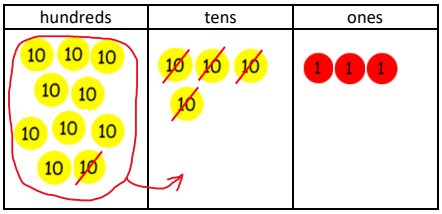
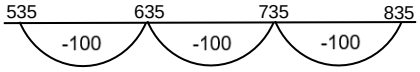
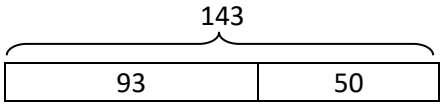
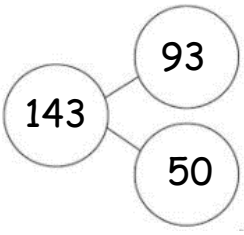


Two-digit - two-digit

$74 - 26 = 74 - 20 - 6$
$= 48$

CALCULATION PROGRESSION: SUBTRACTION

Y3 HTO-O, HTO-T, HTO-H & HTO-HTO

concrete	pictorial	abstract
Three-digit - ones Bead string (for TO-O): *Start at 45, move 5 to jump to the multiple of 10 before, then move the rest.* Base ten blocks: *Make 145, subtract 5 then exchange a ten stick for ten ones to subtract the rest.* 	**Three-digit - ones** Number line – counting back to the multiple of ten first:  Bar model:  Part-part-whole model:  	**Three-digit - ones**  *Partition the 8 in this way because taking away 5 leaves a multiple of ten.* 145 – 8 = 145 – 5 – 3 = 137 *Family of four* 137 + 8 = 145 8 + 137 = 145 145 – 8 = 137 145 – 137 = 8
Three-digit - tens or hundreds Base ten blocks: *Make the starting number. Take away from the tens or hundreds column, exchanging from the next column if needed.*  Place value counters: *Make the starting number. Take away from the tens or hundreds column, exchanging from the next column if needed.*  **Counting:** Count back in tens or hundreds: 143, 133, 123, 113, 103, 93, … 835, 735, 635, 535, …	**Three-digit - tens or hundreds** Drawing base ten:  Number line – counting back in tens or hundreds:  Bar model:  Part-part-whole model:  	**Three-digit - tens or hundreds** 143 – 50 = 93 835 – 300 = 535

Three-digit - two-digit

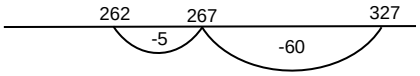
Base ten blocks:

Make the first number, partition the second number and take away the tens then take away the ones.

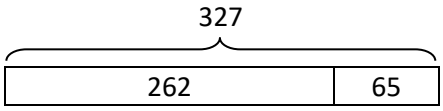
Three-digit - two-digit

Number line:

Partition the second number, add the tens then add the ones.



Bar model:



Three-digit - two-digit

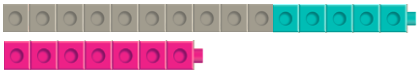
327 – 65 = 327 – 60 – 5
= 262

Finding a difference (including money)

Use this method for comparison problems and when it is more efficient/ easier to count on.

Multilink cubes:

For small numbers, make both numbers and compare them.



Coins:

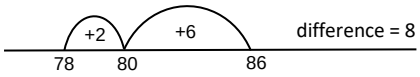
Find change by counting on. Make the cost of the item, then add on coins to reach the amount given.

Finding a difference (including money)

Use this method for comparison problems and when it is more efficient/ easier to count on.

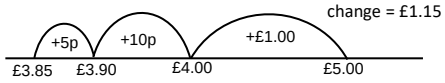
Number line:

Find the difference between numbers that are close in value by counting on to find the amount between them.



Number line:

With money, find change by counting up.



Finding a difference (including money)

Use this method for comparison problems and when it is more efficient/ easier to count on.

86 – 78 = 8

78 + ☐ = 86

£5.00 – £3.85 = £1.15

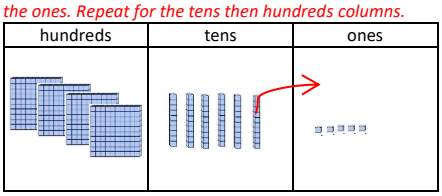
£3.85 + ☐ = £5.00

Three-digit - three-digit column method

Start with no carrying, then carry once, then carry twice.

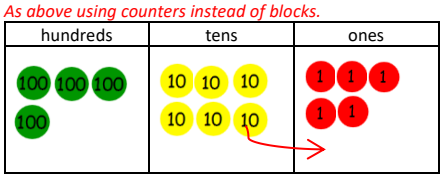
Base ten blocks:

Make the first number. Look at the ones column and ask, "Starting from [the top number], can I take away [the bottom number]?" Exchange if needed then take away the ones. Repeat for the tens then hundreds columns.



Place value counters:

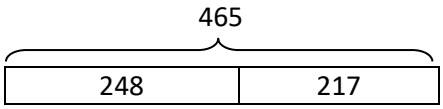
As above using counters instead of blocks.



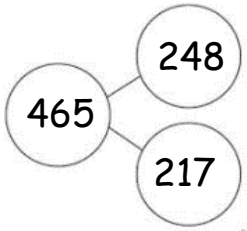
Three-digit - three-digit column method

Start with no carrying, then carry once, then carry twice.

Bar model:



Part-part-whole model:



Three-digit - three-digit column method

Start with no carrying, then carry once, then carry twice.

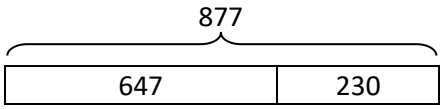
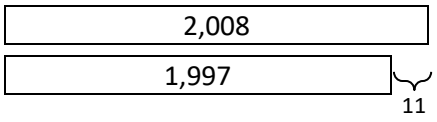
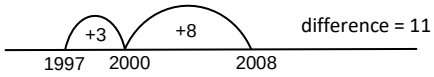
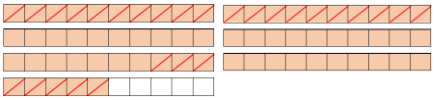
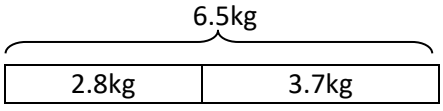
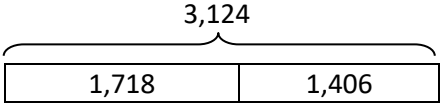
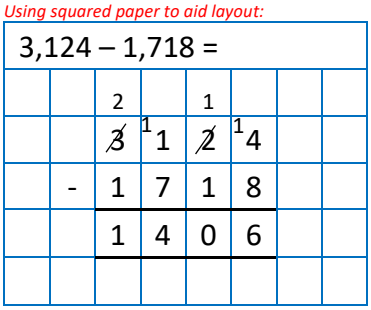
Using squared paper to aid layout:

4	6	5	-	2	4	8	=
			5	15			
		4	~~6~~	~~5~~			
	-	2	4	8			
		2	1	7			

Note: When carrying from one column to the next, the value in <u>both</u> columns needs to be changed.

CALCULATION PROGRESSION: SUBTRACTION

Y4 HTO-HTO, ThHTO-ThHTO & decimals up to 2d.p. (in context)

concrete	pictorial	abstract
<u>Mental calculations</u> **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary), and whether it is more efficient to add or subtract.** Base ten blocks: *To help with the relative size of each column, use blocks on a place value mat to make the first number then take away from the appropriate columns.* Place value counters: *Use place value counters on a place value mat to keep track of when exchanging is needed.*	<u>Mental calculations</u> **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary), and whether it is more efficient to add or subtract.** Bar model:  Comparison bar model:  Number line: 	<u>Mental calculations</u> **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary), and whether it is more efficient to add or subtract.** 3,536 – 1,300 = 3,536 – 1,000 – 300 = 2,236 2,008 – 1,997 = 11 *(count on from 1997)* *Family of four* 1,997 + 11 = 2,008 11 + 2,008 = 1,997 2,008 – 1,997 = 11 2,008 – 11 = 1,997
<u>Decimals</u> **Calculate with decimal tenths in the context of measurement.** Strips of paper: *Cut strips of paper to the length required and place them next to each other to compare them.* Tenth strips:  *Use a strip of ten to represent 1 'whole'. Using several strips, shade in the first number. Cross out the ones, then cross out the tenths.* **Counting:** Count back in decimal tenths from different starting points: 7.4, 7.3, 7.2, 7.1, 7, 6.9, 6.8, …	<u>Decimals</u> **Calculate with decimal tenths in the context of measurement.** Number line – count back the ones, then the tenths:  Bar model: 	<u>Decimals</u> **Calculate with decimal tenths in the context of measurement.** 8.4m – 3.1m = 5.3m 6.5kg – 2.8kg = 3.7kg 10.7ℓ – 2.9ℓ = 7.8ℓ
<u>Column method</u> Base ten blocks: *Use base ten blocks to reinforce the relative size of the digits in different columns. Make the first number. Look at the ones column and ask, "Starting from [the top number], can I take away [the bottom number]?" Exchange if needed then take away the ones. Repeat for the other columns.* Place value counters: *As above using counters instead of blocks.*	<u>Column method</u> Bar model: 	<u>Column method</u> *Using squared paper to aid layout:* 3,124 – 1,718 =  *Note: When carrying from one column to the next, the value in <u>both</u> columns needs to be changed.*

Money

Note:

It is important to think about whether a written method is appropriate. If the amounts are easier to calculate mentally, then a mental method (with or without a number line) should be used.

Coins and notes:

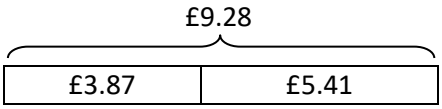
As above using money (could be modelled using blocks or counters instead).

Money

Note:

It is important to think about whether a written method is appropriate. If the amounts are easier to calculate mentally, then a mental method (with or without a number line) should be used.

Bar model:



Money

Note:

It is important to think about whether a written method is appropriate. If the amounts are easier to calculate mentally, then a mental method (with or without a number line) should be used.

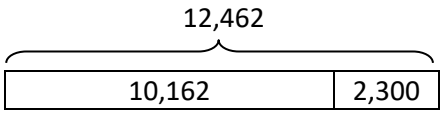
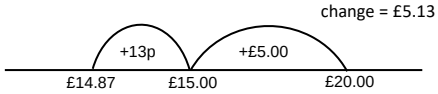
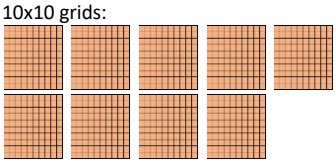
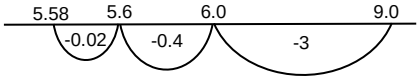
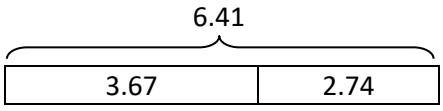
Calculate in pence, then convert to pounds and pence:

£9.28 - £3.87 = £5.41

		8					
		~~9~~ 1	2	8			
	-	3	8	7			
		5	4	1			

CALCULATION PROGRESSION: SUBTRACTION

Y5 Include numbers with more than four digits and decimals up to 2d.p.

concrete	pictorial	abstract
Mental calculations **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary), and whether it is more efficient to add or subtract.** Place value flip book: *Model subtracting from one column by turning that digit on a flip book. Consider what to do when the digit is 0.* 	Mental calculations **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary), and whether it is more efficient to add or subtract.** Bar model:  Number line: *With money, find change by counting up.* 	Mental calculations **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary), and whether it is more efficient to add or subtract.** 12,462 – 2,300 = 12,462 – 2,000 – 300 = 10,162 £20.00 – £14.87 = £5.13 £14.87 + ☐ = £20.00
Mental calculations (decimals) **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary).** 10x10 grids:  *Using a 10x10 grid as 1 'whole', shade in the numbers in the first number. Cross out the ones, then cross out the decimal parts.*	Mental calculations (decimals) **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary).** Number line – count back the ones, then the decimal parts:  Bar model: 	Mental calculations (decimals) **Make decisions about when it is appropriate to calculate mentally (with jottings if necessary).** 9 – 3.42 = 9.0 – 3 – 0.4 – 0.02 = 5.58 *Family of four* 3.67 + 2.74 = 6.41 2.74 + 3.67 = 6.41 6.41 – 3.67 = 2.74 6.41 – 2.74 = 3.67
Column method	Column method	Column method *Using squared paper to aid layout:* 64,583 – 7,286 = 5 4 17 6 14 5 8 13 - 7 2 8 6 5 7 2 9 7
Column method (decimals) **Same number of decimal places.** Place value counters: *Use decimal place value counters to model subtraction of decimals, exchanging where needed.*	Column method (decimals) **Same number of decimal places.**	Column method (decimals) **Same number of decimal places.** *Stress the importance of lining up the decimal points.* 68.04 – 14.78 = 7 9 6 8 . 10 14 - 1 4 . 7 8 5 3 . 2 6

CALCULATION PROGRESSION: SUBTRACTION

Y6 Include numbers with more than four digits and decimals

concrete	pictorial	abstract
Practise the skills from Y5 with larger numbers and decimals with differing numbers of decimal places, including some numbers that contain a 0. Practise making decisions about when it is more appropriate to use a mental method, and whether to add or subtract.		
<u>Column method (decimals)</u> **Different number of decimal places.** Place value counters: *Use decimal place value counters to model subtraction of decimals, exchanging where needed.*	<u>Column method (decimals)</u> **Different number of decimal places.**	<u>Column method (decimals)</u> **Different number of decimal places.** *Stress the importance of lining up the decimal points and add zeros as place holders so both numbers have the same number of decimal places.* 70.74 – 4.548 = 6 6 13 ~~7~~ 10 . ~~7~~ ~~4~~ 10 + 4 . 5 4 8 6 6 . 1 9 2